ECS455 Chapter 2

Cellular Systems

2.4 Traffic~~e~~ "Handling Capacity" and Erlang B Formula

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Capacity Concept: A Revisit

- Q: If I have *m* channels per cell, is it true that my cell can support only *m* user?
- A: Yes and No
- Let's try one example.
- How often do you make a call?
  - 3 calls a day, on average. ← λ_u ← user
- How long is the call?
  - 10 mins (per call), on average. ← $H = \frac{1}{\mu}$
- So, one person uses

$$3\ \frac{\cancel{\text{calls}}}{\text{day}} \times \frac{10\ \text{mins}}{\cancel{\text{call}}} = \frac{30\ \text{mins}}{\text{day}} = \frac{\cancel{30}\ \cancel{\text{mins}}}{24 \times \cancel{60}_{2}\ \cancel{\text{mins}}} = \frac{1}{48}\ [\text{Erlang}]$$

Capacity Concept: A Revisit

- If we can "give" the time that "User 1" is idle to other users,
  - then one channel can support 48 users!!

    (48x "capacity")
- True? (Not quite)

- 48 users is possible if we have a way to manipulate all 48 users to not make calls when another user is using the channel.

- Real users access the channel randomly. (Call initiation/request times are random.)

- If we allow >1 users, then we (the users) will have to deal with congestion.

New Concepts

- Using m as the capacity of a cell is too small.
- We can let more than one user share a channel by using it at different times.
- The number of users that a cell can support can then exceed m.
- Call initiation times are random
- **Blocked calls**
- Probability of (call) blocking P_b
  - the likelihood that a call is blocked because there is no available channel.
  - 1%, 2%, 5%

Trunking

- Allow a large number (n) of users to **share** the relatively small number of channels in a cell (or a sector) by providing access to each user, **on demand**, from a **pool** of available channels.
- Exploit the **statistical behavior** of users.
- Each user is allocated a channel on a per call basis, and upon termination of the call, the previously occupied channel is immediately returned to the pool of available channels.

Common Terms (1)

- **Traffic Intensity**: Measure of channel time utilization (traffic load / amount of traffic), which is the average channel occupancy measured in **Erlangs**. In our example, one user utilizes $A_u = \frac{1}{48}$ Erlang
  - Dimensionless
  - Denoted by A. If we have $n=10$ users in the pool, then they utilize $A = \frac{10}{48}$ Erlang.
- **Holding Time**: Average duration of a typical call.
  - Denoted by $\mathbf{H = 1/\mu}$. $= 10$ mins
- **Request Rate**: The average number of call requests per unit time. Denoted by λ. $\lambda_u = 3$ calls/day $\lambda = 10 \times 3 = 30$ calls/day.
- Use $\mathbf{A_u}$ and λ_u to denote the corresponding quantities for one user.
- Note that $A = nA_u$ and $\lambda = n\lambda_u$ where n (10) is the number of users supported by the pool (trunked channels) under consideration.

Common Terms (2)

- **Blocked Call**: Call which cannot be completed at time of request, due to congestion. Also referred to as a **lost call**.
- **Grade of Service (GOS)**: A measure of congestion which is specified as the probability of a call being blocked (for Erlang B).

  P_b $\quad$ $P_b \leq 0.02$

  - The AMPS cellular system is designed for a GOS of 2% blocking. This implies that the channel allocations for cell sites are designed so that 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.

M/M/m/m Assumption

- **Blocked calls cleared**
  - Offers no queuing for call requests.
  - For every user who requests service, it is assumed there is no setup time and the user is given immediate access to a channel if one is available.
  - If no channel~~s~~ are available, the requesting user is blocked without access and is free to try again later.
- **Calls arrive as determined by a *Poisson process*.**
- There are memoryless arrivals of requests, implying that all users, including blocked users, may request a channel at any time.
- There are an infinite number of users (with finite overall request rate).
  - The finite user results always predict a smaller likelihood of blocking. So, assuming infinite number of users provides a conservative estimate.

length of tu call

- **The duration of the time that a user occupies a channel is *exponentially distributed***, so that longer calls are less likely to occur.
- There are *m* channels available in the trunking pool.
  - For us, m = the number of channels for a cell ~~(C)~~ or for a sector

Erlang B Formula

$$P_b = \frac{\frac{A^m}{m!}}{\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

m = Number of trunked channels

Call blocking probability

$A =$ traffic intensity or load [Erlangs]

$$= \frac{\lambda}{\mu}$$

λ = Average # call attempts/requests per unit time

$$\frac{1}{\mu} = H = \text{Average call length}$$

$= \lambda \times \frac{1}{\mu}$

$= \lambda \times H$

In MATLAB, use `erlangb(m,A)`

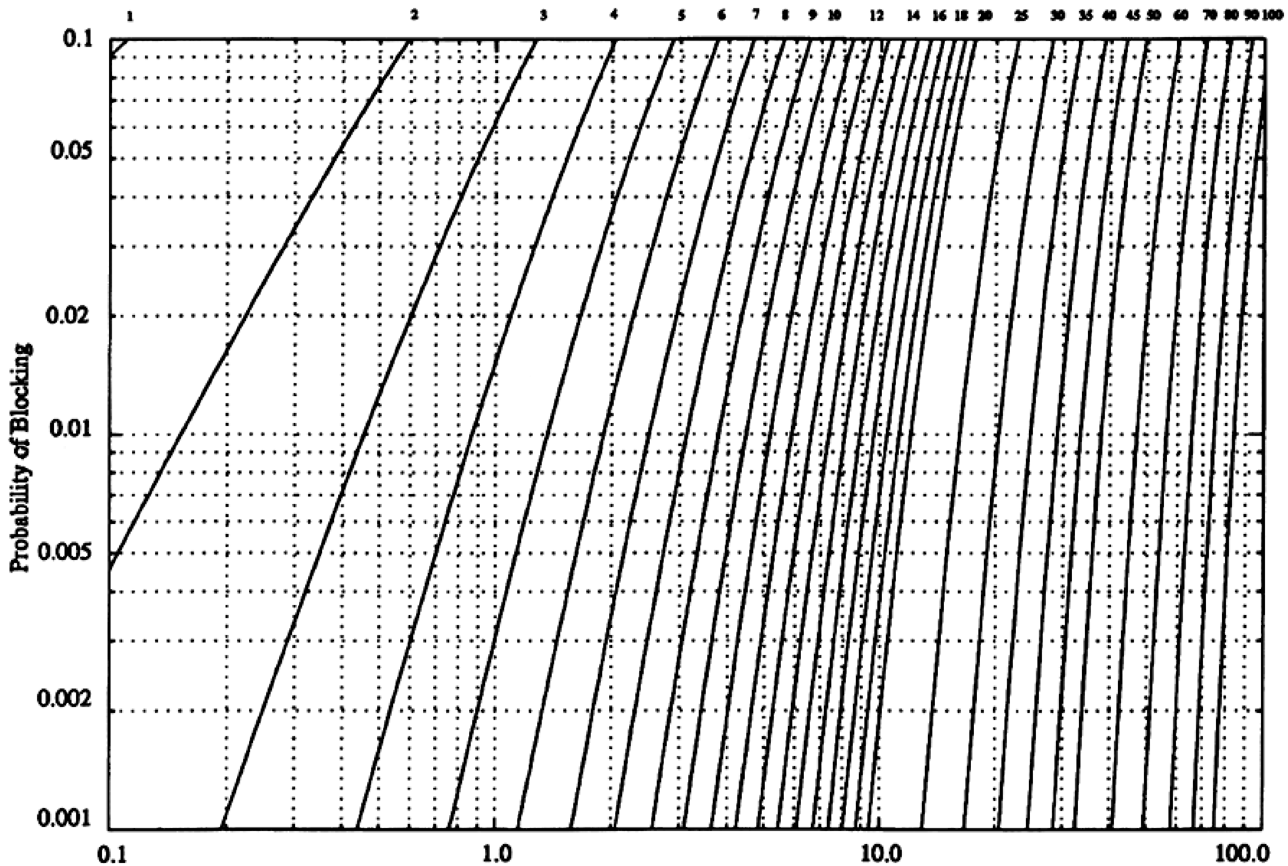
not a build-in function.

You can download the .m file.

Erlang B Formula and Chart

$$P_b = \frac{\dfrac{A^m}{m!}}{\displaystyle\sum_{i=0}^{m} \frac{A^i}{i!}}.$$

Number of Trunked Channel (m)



Traffice Intensity in Erlangs (A)

(log-log plot)

Example 1

$P_b \le 0.005$

- How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system?

  (a) 5 $m = 5$

  (b) 10 $m = 10$

- Assume each user generates $A_u = 0.1$ Erlangs of traffic.

$A_u = \lambda_u \times \frac{1}{\mu}$

For example,

6 times/day

average 24 min

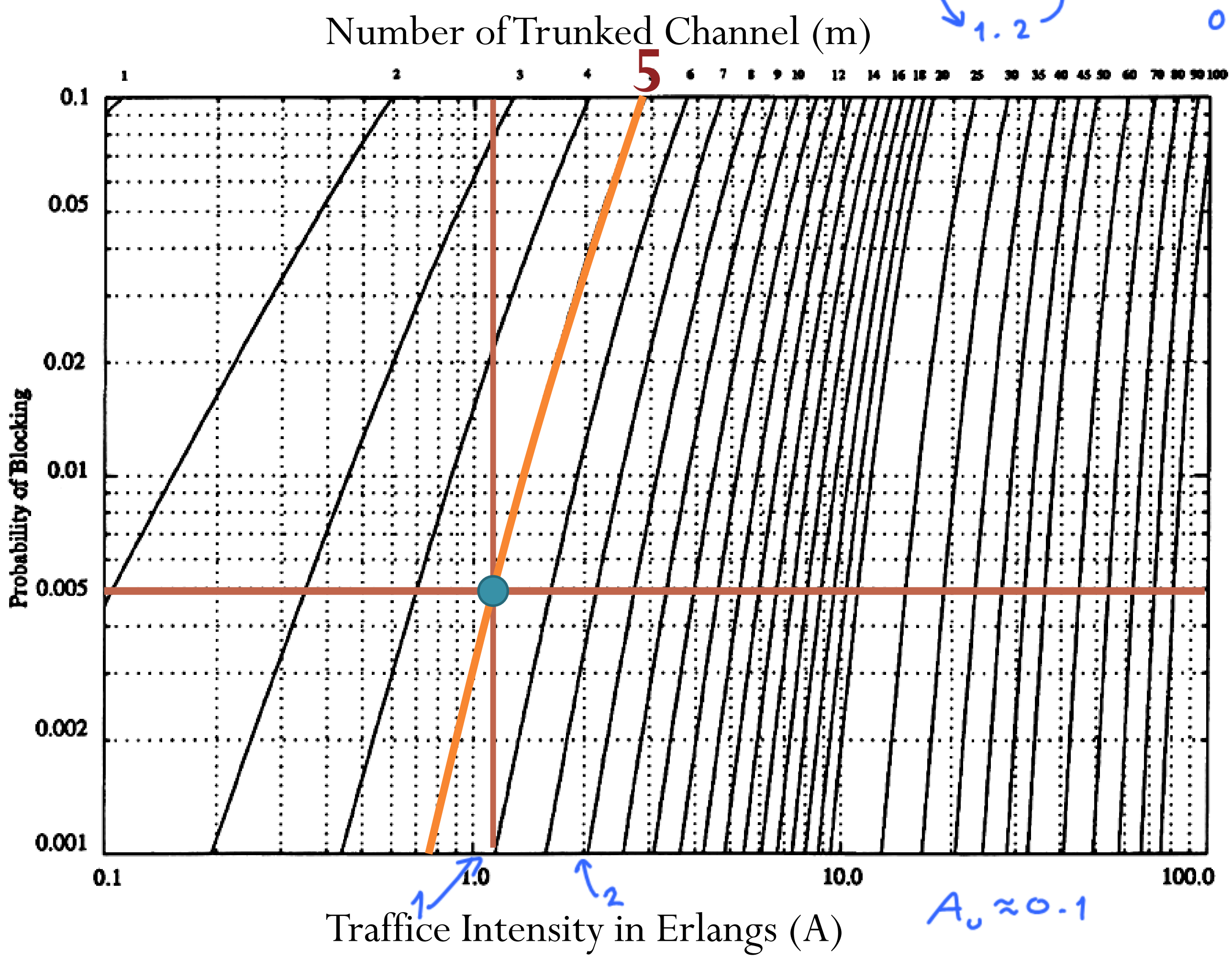
$\Rightarrow \frac{6}{\cancel{24} \times 60} \times \cancel{24} = \frac{1}{10}$

$m = 5 \xrightarrow{\text{MATLAB}} A = 1.13 \longrightarrow 11$ users

Example 1a

$m = 5$

A	P_b
1.1	0.0045
1.13	0.0050
1.14	0.0051
1.15	0.0053
1.2	0.0063



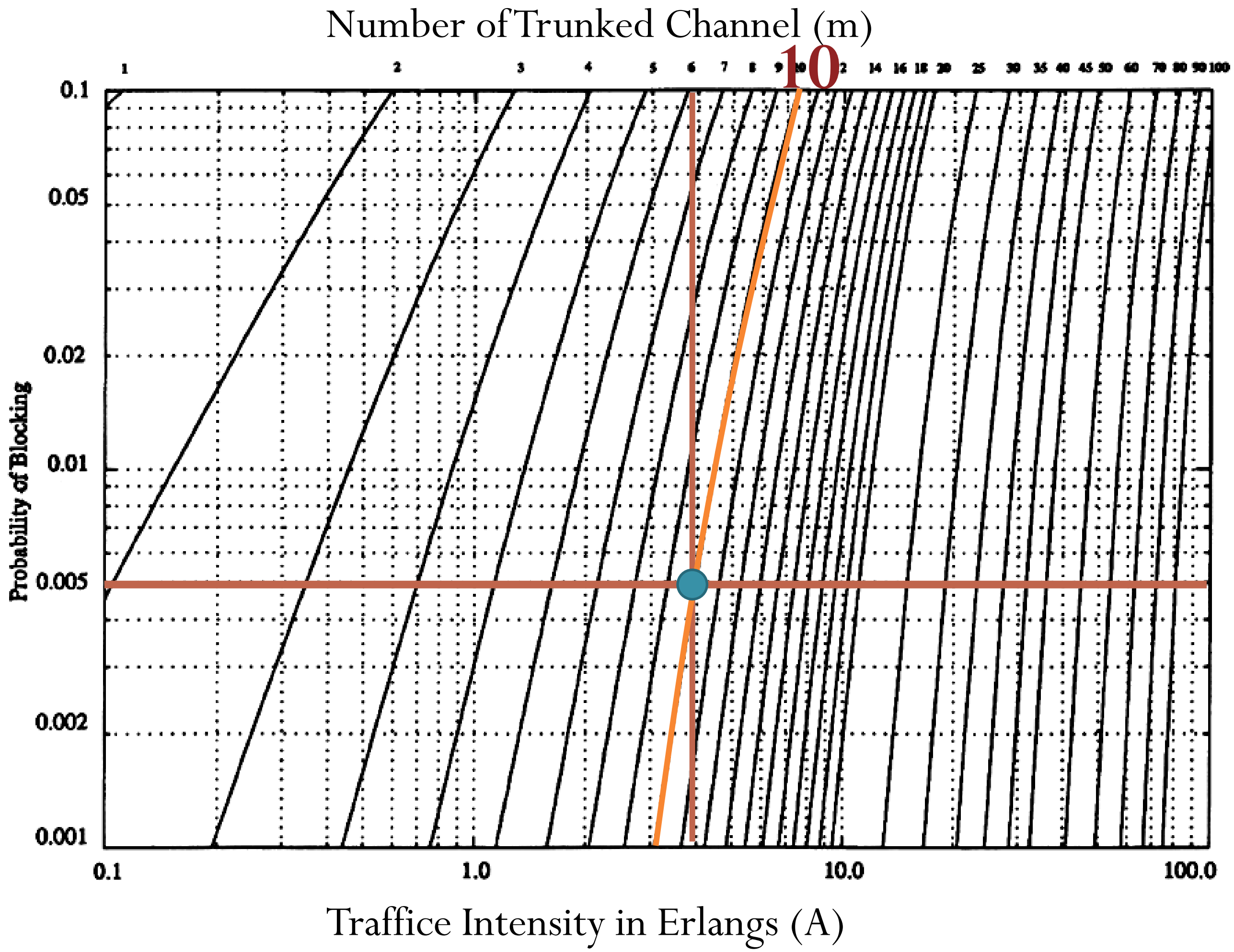
$A_u \approx 0.1$

$A = nA_u$

$A \approx 1 \Rightarrow n \approx 10$ users

Example 1b

m = 10 ⟶ A = 3.96 ⟶ 39 users



$A \approx 4 \Rightarrow n \approx 40$ users

Example 2.1

- Consider a cellular system in which
  - an average call lasts two minutes $H = \frac{1}{\mu} = 2$ mins
  - the probability of blocking is to be no more than 1%. $P_b \leq 0.01$
- If there are a total of 395 traffic channels for a seven-cell ($N = 7$) reuse system, there will be about 57 ($395/7 \approx$; m) traffic channels per cell.
- From the Erlang B formula, can handle 44.2 Erlangs or **1326 calls per hour**.

$$A = \lambda \times \frac{1}{\mu}$$

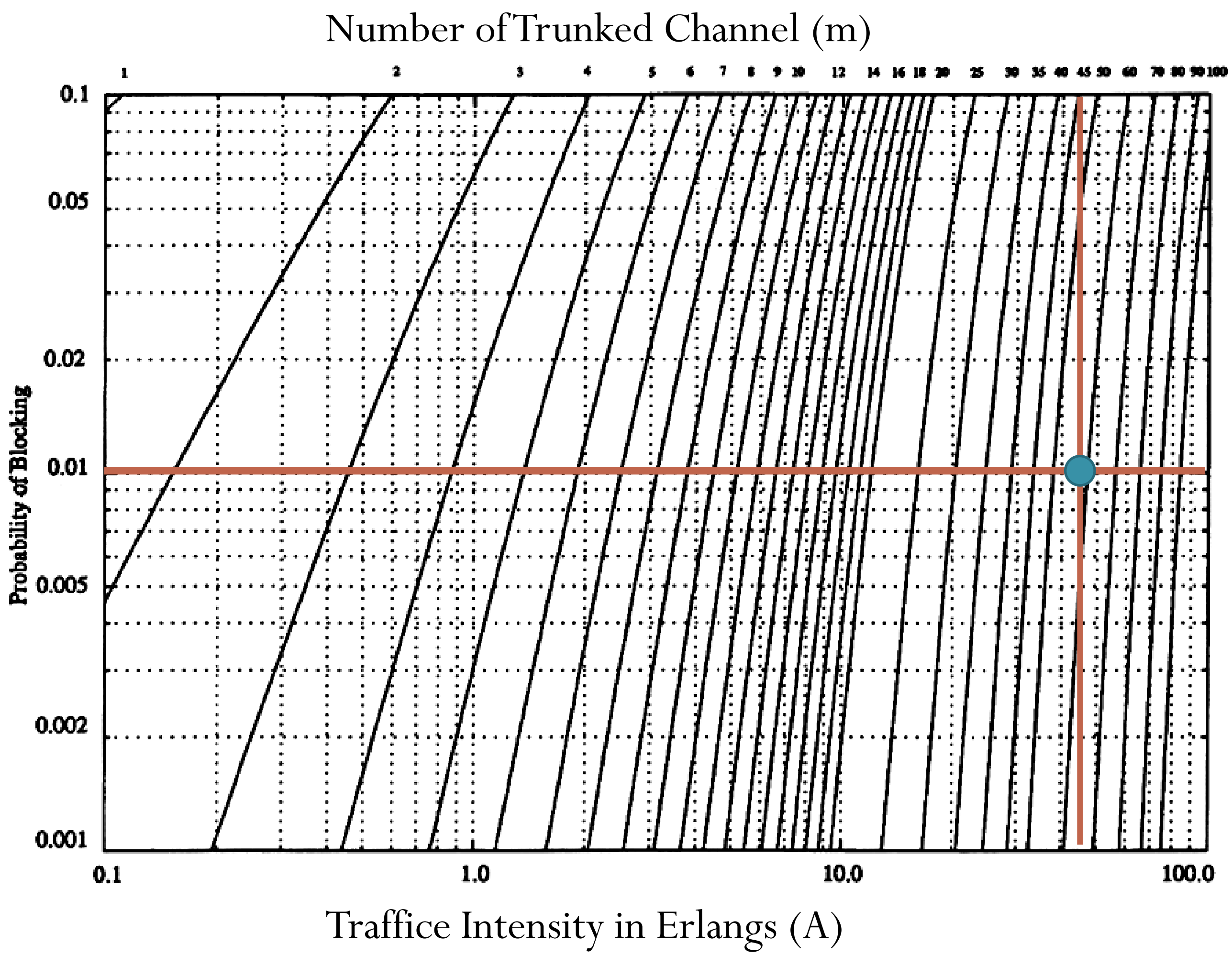
$$44.2 = \lambda \times \frac{2 \text{ mins}}{\text{call}}$$

$$\lambda = \frac{44.2}{2} \frac{\text{calls}}{\text{min}} = 22.1 \text{ calls/min}$$

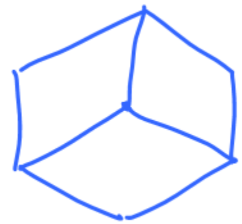
$\times 60$

[Rappaport, 2002, Ex 3.9, p 92]

Example 2.1: Erlang B



Example 2.2



- Now employing **120° sectoring**, there are only 19 channels per sector (57/3 ~~antennas~~ channel).

  m

  $P_b \leq 0.01$

- For the same probability of blocking and average call length, each sector can handle 11.2 Erlangs or 336 calls per hour.
- Since each cell consists of three sectors, this provides a cell capacity of $3 \times 336 =$ **1008 calls per hour**, which amounts to a 24% decrease when compared to the unsectored case.
- Thus, sectoring decreases the **trunking efficiency** while improving the SIR for each user in the system.

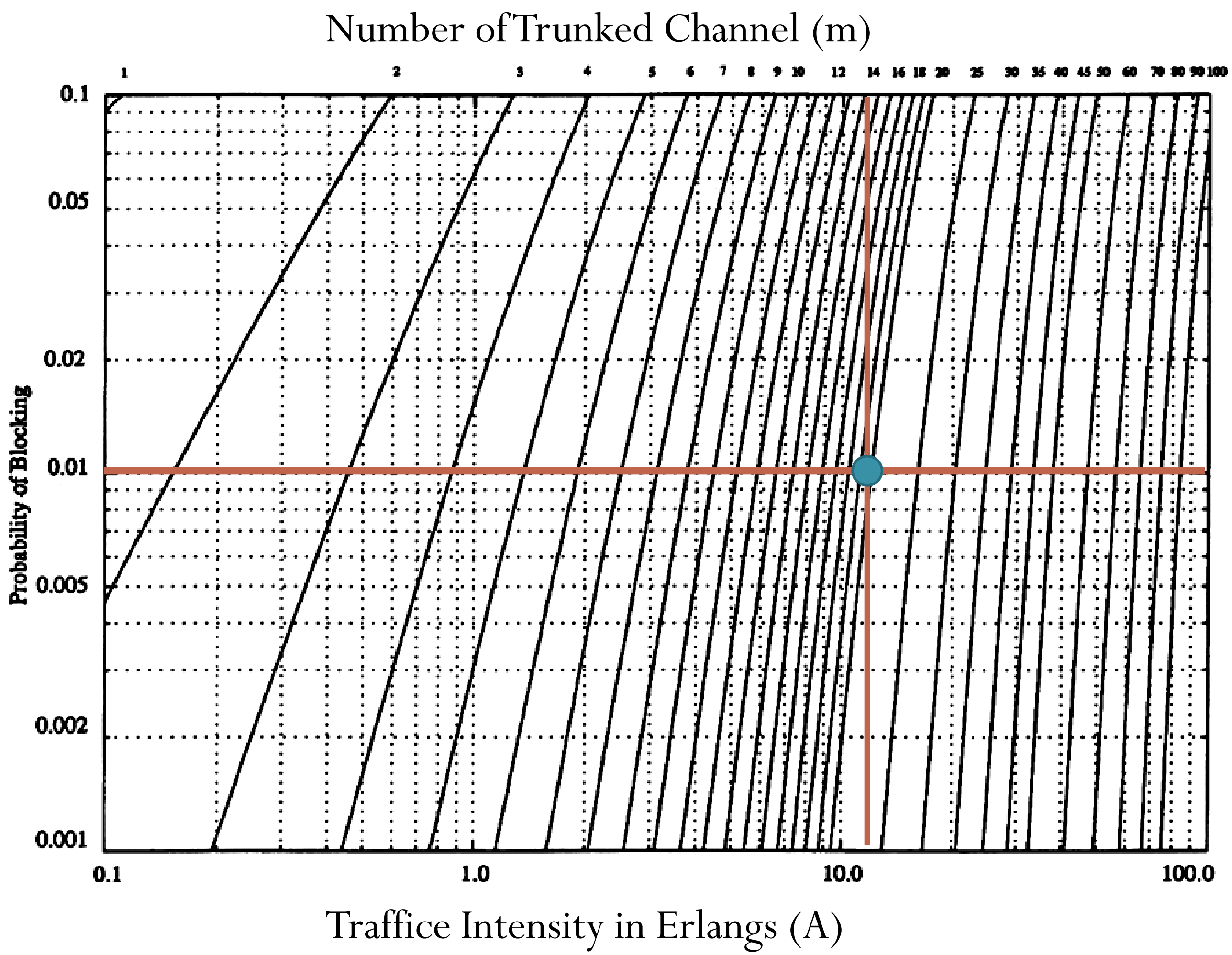
worse Erlang

but

better SIR.

[Rappaport, 2002, Ex 3.9, p 92]

Example 2.2: Erlang B



Erlang B Trunking Efficiency

Table 3.4 Capacity of an Erlang B System

Number of Channels C m	1% **= 0.01**	**Capacity (Erlangs) for GOS** P_b **= 0.005**	**= 0.002**	0.1% **= 0.001**
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10 ×2	4.46 ×>2	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

m

A

[Rappaport, 2002, Table 3.4]

Summary of Chapter 2: Big Picture

S = total # available duplex radio channels for the system

Frequency reuse with **cluster size N**

Path loss exponent

"Capacity" $C = \dfrac{A_{\text{total}}}{A_{\text{cell}}} \times \dfrac{S}{N}$

Tradeoff

$$\frac{S}{I} \approx \frac{kR^{-\gamma}}{K \times \left(kD^{-\gamma}\right)} = \frac{1}{K}\left(\frac{D}{R}\right)^{\gamma} = \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

m = # channels allocated to each cell.

Omni-directional: K = 6
120° Sectoring: K = 2
60° Sectoring: K = 1

Trunking

m = # trunked channels

λ = Average # call attempts/requests per unit time

Call blocking probability $P_b = \dfrac{\dfrac{A^m}{m!}}{\displaystyle\sum_{i=0}^{m} \dfrac{A^i}{i!}}$.

A = traffic intensity or load [Erlangs] $= \dfrac{\lambda}{\mu}$

Erlang-B formula

$\dfrac{1}{\mu} = H$ = Average call length

Example 3 (1)

- System Design
- 20 MHz of total spectrum.
- Each simplex channel has 25 kHz RF bandwidth.
- The number of duplex channels:

$$S = \frac{20\times10^6}{2\times25\times10^3} = 400 \text{ channels}$$

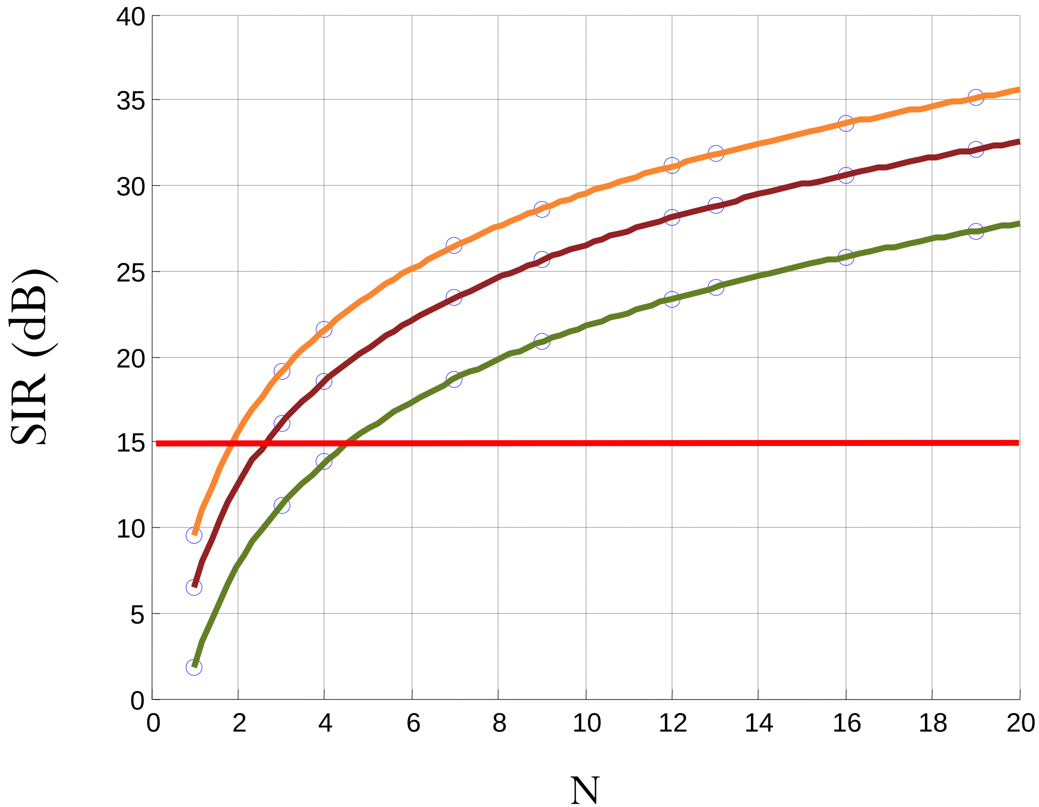
- $\gamma = 4$
- Design requirements:
  - SIR $\geq$ 15 dB
  - $P_b \leq 5\%$

Example 3 (2)

- SIR ≥ 15 dB

$$\frac{S}{I} \approx \frac{1}{K}\left(\sqrt{3N}\right)^{\gamma}$$

```
clear all; close all;
y = 4;
figure; grid on; hold on;
for K = [1,2,6]
    N = [1, 3, 4, 7, 9, 12, 13, 16, 19];
    SIR = 10*log10(1/K*((sqrt(3*N)).^y))
    plot(N,SIR,'o')
    N = linspace(1,20,100);
    SIR = 10*log10(1/K*((sqrt(3*N)).^y));
    plot(N,SIR)
end
```



60° sectoring

K = 1 → N = 3

120°

K = 2 → N = 3

omni

K = 6 → N = 7

Example 3 (3)

		Omnidirectional	Sectoring (120°)	Sectoring (60°)
	K	6	2	1
	N	7	3	3
	SIR [dB]	18.7	16.1	19.1
$S=400$	#channels/cell	400/7 = 57	400/3 = 133	400/3 = 133
	#sectors	1	3	6
$m=$	#channels/sector	57	133/3 = 44	133/6 = 22
	A [Erlangs]/sector	51.55	38.56 best	17.13
	A [Erlangs]/cell	51.55	38.56×3 = 115.68	17.13×6 = 102.78
	#users/cell	18558	41645	37001

Assume that each user makes 2 calls/day and 2 min/call on average → 1/360 Erlangs.

Make sure that you understand where numbers in this table come from!